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LINEAR ACCELERATION GUIDANCE SCHEME FOR LANDING AND LAUNCH TRAJECTORIES IN A VACUUM

by Victor R. Bond

Manned Spacecraft Center

Houston, Texas

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LINEAR ACCELERATION GUIDANCE SCHEME FOR LANDING AND LAUNCH TRAJECTORIES IN A VACUUM

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SUMMARY

Guidance equations are developed for guiding a spacecraft with finite continuous thrust from an initial position and velocity to a terminal position and velocity in a vacuum. The solution to this two-point boundary-value problem is obtained by assuming that the change in altitude and out-of-plane distance during the motion is small compared with the initial radius from the center of the attracting body to the spacecraft, and by prescribing linear acceleration for each of the three acceleration components. The resulting thrust, in general, is variable, but by introduction of a constraining relation, constant-thrust trajectories can be generated. Similarly, by introduction of another constraining relation, constant-pitch-angle trajectories may be generated. The guidance equations obtained may be used to guide a spacecraft to a landing on the moon, or to guide a spacecraft during launching from the moon. A comparison of these results with an optimum trajectory shows that the guidance equations yield a near-optimum trajectory for the case of a range-free, constant-thrust landing maneuver to a point near the lunar surface.

INTRODUCTION

When a spacecraft, during lunar landing, launch, or abort, is to be guided to a specified set of terminal conditions so that fuel usage is near minimum, a set of guidance equations must be mechanized onboard the spacecraft that will predict, at any time, the necessary thrust vector. In obtaining a solution for the guidance equations, it is desirable that the following criteria be met: The equations must be computationally simple; they must be suitable for use in as many operational modes as possible; and they must yield a solution that is near the optimum in fuel usage.

In order to obtain such a set of guidance equations, the first step is choosing a suitable approximation which would render the equations of motion amenable to closed form solution. To simplify the equations, the change in spacecraft altitude and out-of-plane distance is assumed to be small compared with the initial radius from the center of the attracting body. This assumption is similar to the assumption of a uniform gravitational field. The next step is to solve the resulting simplified two-point boundary-value problem explicitly.

In this paper, the approach taken in solving the problem is to prescribe that the radial, circumferential, and out-of-plane components of the thrust acceleration are continuous and vary linearly with time. These approximate equations of motion can then be solved in closed form. Each of the linear acceleration components involves two parameters: One is the initial level of the applied acceleration, and the other is the rate of change of the applied acceleration. Since there are three acceleration components, a total of six parameters are introduced which can be determined in closed form in terms of the six specified terminal conditions. The equations for these parameters constitute the guidance equations which will always insure that the spacecraft's trajectory will meet the specified terminal conditions. By allowing one of the acceleration parameters to vanish, the terminal range is unconstrained, and only five specified terminal conditions are met. The pitch-angle control law that results from this acceleration scheme is identical in form to the bilinear tangent law which is shown in reference 1 to be optimum without regard to boundary conditions or thrust-magnitude history. The assumptions made in reference 1 are that the gravitational field is uniform and that there are no aerodynamic or other dissipative forces.

The approach taken in reference 2 is to assume that the acceleration components are linear (in time) plus a gravity component. When the assumed accelerations are substituted back into the equations of motion, the gravity terms cancel completely. The work in this paper and in reference 2 was carried out independently and concurrently. Reference 2 does not introduce the constant-pitch-angle constraint.

Solutions to this two-point boundary-value problem, in general, require variable thrust. If the condition that the thrust remain constant is imposed, a relation may be introduced from which a burning time that will insure constant thrust may be calculated. Similarly, if the condition that the pitch angle remain constant is imposed, a second relation may be introduced from which a burning time that will insure a constant pitch angle may be calculated.

SYMBOLS

A	magnitude of acceleration vector
$\vec{A}$	acceleration vector
a, b, c	parameters in acceleration specification defined by equations (25)
a_r, a_ϕ, a_z	linear acceleration component in radial, circumferential, and out-of-plane directions, respectively
C_1, C_2, C_3, C_4, C_5	time coefficients in the altitude and altitude rate equations defined by equations (16)

g_e	gravity at surface of earth, 32.1849 ft/sec^2
h	altitude
I_{sp}	specific impulse
m	mass
R	radius of attracting body
r, ϕ, z	cylindrical coordinates defined in figure 1
$\vec{u}_r, \vec{u}_\phi, \vec{u}_z$	unit vectors defined in figure 1
T	thrust
t	time
W	weight
X	arc length or range along surface of attracting body
$\alpha_r, \alpha_\phi, \alpha_z$	initial level of applied acceleration in radial, circumferential, and out-of-plane directions, respectively
β	thrust azimuth angle defined in figure 1
θ	thrust pitch angle defined in figure 1
μ	universal gravitational constant times mass of attracting body
ρ	magnitude of $\vec{\rho}$
$\vec{\rho}$	position vector of spacecraft, $r\vec{u}_r + z\vec{u}_z$
τ	burning time
$\Psi_r, \Psi_\phi, \Psi_z$	rate of change of applied acceleration in radial, circumferential, and out-of-plane directions, respectively
$(\dot{\ })$	derivative of ( ) with respect to t

Subscripts:

0	value of variable when $t = 0$
r	pertaining to initial radial direction

z	pertaining to z direction
τ	value of variable when $t = \tau$
ϕ	pertaining to circumferential, or X, direction

DERIVATION OF EQUATIONS

Solution to the Two-Point Boundary-Value Problem

The equations of motion for a thrusting spacecraft in a gravitational field are

$$\ddot{r} - r\dot{\phi}^2 + \frac{\mu r}{\rho^3} = a_r \tag{1}$$

$$\frac{d}{dt}\left(r^2\dot{\phi}\right) = ra_\phi \tag{2}$$

$$\ddot{z} + \frac{\mu z}{\rho^3} = a_z \tag{3}$$

where

$$\rho = \left(r^2 + z^2\right)^{\frac{1}{2}}$$

The coordinates r, ϕ, and z are the cylindrical coordinates of the spacecraft as shown in figure 1.

The equations (1), (2), and (3) may be solved analytically by prescribing the linear acceleration components

$$a_r = A \sin\theta \cos\beta = \alpha_r + \Psi_r t \tag{4}$$

$$a_\phi = A \cos\theta \cos\beta = \alpha_\phi + \Psi_\phi t \tag{5}$$

$$a_z = A \sin\beta = \alpha_z + \Psi_z t \tag{6}$$

and making additional assumptions regarding the smallness of z and the change in r compared with the

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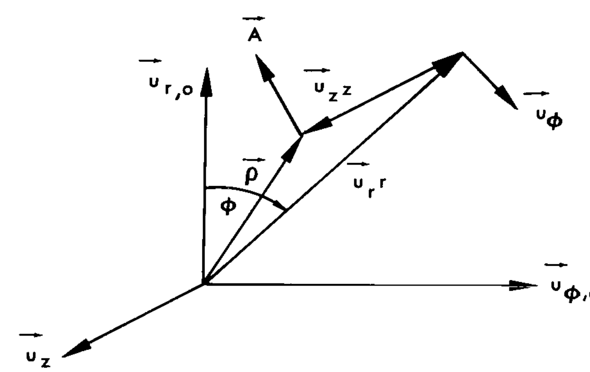
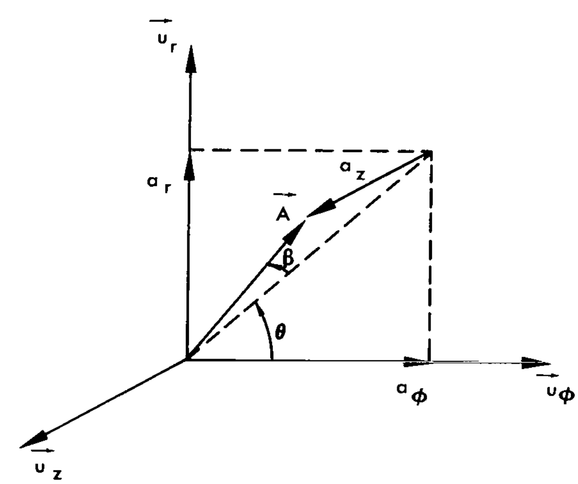


(a) Cylindrical coordinate system (r, ϕ, z) in which motion is described



(b) Orientation of thrust vector

Figure 1.- Coordinate system

initial radius r_0, and regarding the magnitude of the applied acceleration terms.

Let the altitude be defined by $h = r - R$, and assume that during the motion r changes only slightly from its initial value r_0, so that in equation (1), r may be replaced by r_0, and $\ddot{r}$ by $\ddot{h}$. Also, assume that $z \ll r_0$ and may be neglected. Then equation (1) becomes

$$\ddot{h} - r_0 \dot{\phi}^2 + \frac{\mu}{r_0^2} = \alpha_r + \Psi_r t \tag{7}$$

Making the same assumptions as above, equation (2) becomes

$$2\dot{h}\dot{\phi} + r_0\ddot{\phi} = \alpha_\phi + \Psi_\phi t$$

The first term on the left is the Coriolis acceleration which arises due to the motion of the spacecraft relative to the rotating axis system defined by the unit vectors $\vec{u}_r$, $\vec{u}_\phi$, and $\vec{u}_z$. This term will usually be small compared with the applied acceleration a_ϕ and will be neglected. With this assumption and by introducing the surface range $X = R\phi$, equation (2) finally reduces to

$$\ddot{X} = \frac{R}{r_0}\left(\alpha_\phi + \Psi_\phi t\right) \tag{8}$$

Equation (3) becomes

$$\ddot{z} = \alpha_z + \Psi_z t \tag{9}$$

if it is assumed that z is very small so that the gravity term is negligible compared with the applied acceleration a_z.

The assumptions made in deriving equations (7), (8), and (9) are not as stringent as those for the usual flat-body analysis which neglects both centrifugal and Coriolis accelerations. In the present analysis, only the Coriolis acceleration is neglected. The centrifugal acceleration is retained by the inclusion of the $- r_0\dot{\phi}^2$ term in equation (7).

The differential equations, (7), (8), and (9), may now be integrated to give the equations for the three velocity and the three position components.

$$\dot{h} = \dot{h}_0 + C_1 t + \frac{1}{2} C_2 t^2 + \frac{1}{3} C_3 t^3 + \frac{1}{4} C_4 t^4 + \frac{1}{5} C_5 t^5 \tag{10}$$

$$\dot{X} = \dot{X}_0 + \frac{R}{r_0}\left(\alpha_\phi t + \frac{1}{2} \Psi_\phi t^2\right) \tag{11}$$

$$\dot{z} = \dot{z}_0 + \alpha_z t + \frac{1}{2} \Psi_z t^2 \tag{12}$$

$$h = h_0 + \dot{h}_0 t + \frac{1}{2} C_1 t^2 + \frac{1}{6} C_2 t^3 + \frac{1}{12} C_3 t^4 + \frac{1}{20} C_4 t^5 + \frac{1}{30} C_5 t^6 \tag{13}$$

$$X = X_0 + \dot{X}_0 t + \frac{R}{r_0}\left(\frac{1}{2} \alpha_\phi t^2 + \frac{1}{6} \Psi_\phi t^3\right) \tag{14}$$

$$z = z_0 + \dot{z}_0 t + \frac{1}{2} \alpha_z t^2 + \frac{1}{6} \Psi_z t^3 \tag{15}$$

where $\dot{h}_0$, $\dot{X}_0$, $\dot{z}_0$, h_0, X_0, and z_0 are specified values of the velocity and position components when $t = 0$; and

$$\left.\begin{aligned}
C_1 &= \frac{r_0}{R^2} \dot{X}_0^{\,2} - \frac{\mu}{r_0^{\,2}} + \alpha_r \\
C_2 &= \frac{2\dot{X}_0 \alpha_\phi}{R} + \Psi_r \\
C_3 &= \frac{\alpha_\phi^{\,2}}{r_0} + \frac{\dot{X}_0}{R} \Psi_\phi \\
C_4 &= \frac{\alpha_\phi}{r_0} \Psi_\phi \\
C_5 &= \frac{\Psi_\phi^{\,2}}{4r_0}
\end{aligned}\right\} \tag{16}$$

The solution is now determined for the approximate problem except for the acceleration parameters, α_ϕ, Ψ_ϕ, α_r, Ψ_r, α_z, and Ψ_z, which determine the thrust history, that is, the thrust magnitude and direction. The six equations of motion, equations (10) to (15), may now be solved for these parameters in terms of the terminal conditions, where $t = \tau$, $X = X_\tau$, $\dot{X} = \dot{X}_\tau$, $h = h_\tau$, $\dot{h} = \dot{h}_\tau$, $z = z_\tau$, and $\dot{z} = \dot{z}_\tau$. After considerable manipulation, the equations for the acceleration parameters become

$$\alpha_\phi = -\frac{2}{\tau}\frac{r_0}{R}\left[\frac{3}{\tau}\left(X_0 - X_\tau\right) + 2\dot{X}_0 + \dot{X}_\tau\right] \tag{17}$$

$$\Psi_\phi = \frac{6}{\tau^2}\frac{r_0}{R}\left[\frac{2}{\tau}\left(X_0 - X_\tau\right) + \dot{X}_0 + \dot{X}_\tau\right] \tag{18}$$

$$\alpha_r = \frac{\mu}{r_0^2} - \frac{r_0}{R^2}\dot{X}_0^2 + \frac{1}{\tau}\left(\dot{h}_\tau - \dot{h}_0\right) - \frac{C_2}{2}\tau - \frac{C_3}{3}\tau^2 - \frac{C_4}{4}\tau^3 - \frac{C_5}{5}\tau^4 \tag{19}$$

$$\Psi_r = -2\alpha_\phi\frac{\dot{X}_0}{R} + \frac{12}{\tau^3}\left(h_0 - h_\tau\right) + \frac{6}{\tau^2}\left(\dot{h}_0 + \dot{h}_\tau\right) - C_3\tau - \frac{9}{10}C_4\tau^2 - \frac{4}{5}C_5\tau^3 \tag{20}$$

$$\alpha_z = -\frac{2}{\tau}\left[\frac{3}{\tau}\left(z_0 - z_\tau\right) + 2\dot{z}_0 + \dot{z}_\tau\right] \tag{21}$$

$$\Psi_z = \frac{6}{\tau^2}\left[\frac{2}{\tau}\left(z_0 - z_\tau\right) + \dot{z}_0 + \dot{z}_\tau\right] \tag{22}$$

For a completely specified set of initial and terminal conditions, the simplified boundary value problem is now completely solved. The only free parameter remaining is the burning time which may be arbitrarily chosen within operational constraints.

Constant-Thrust Constraint

The preceding analysis yields a thrust history that is, in general, variable. It can be shown that a solution to the special case of constant-thrust magnitude may be obtained by proper choice of the burning time and the thrust level. The acceleration for constant thrust (linearly varying mass) is

$$A = \frac{T}{m_0 - \frac{T}{g_e I_{sp}} t} = \frac{T/m_0}{1 - \frac{T/m_0}{g_e I_{sp}} t} \tag{23}$$

The total acceleration using the linear acceleration components given by equations (4), (5), and (6) is of the form

$$A(\tau) = \left[a(\tau) + b(\tau)t + c(\tau)t^2\right]^{\frac{1}{2}} \tag{24}$$

where

$$\left.\begin{aligned} a(\tau) &= \alpha_\phi^2(\tau) + \alpha_r^2(\tau) + \alpha_z^2(\tau) \\ b(\tau) &= 2\left[\alpha_\phi(\tau)\ \Psi_\phi(\tau) + \alpha_r(\tau)\ \Psi_r(\tau) + \alpha_z(\tau)\ \Psi_z(\tau)\right] \\ c(\tau) &= \Psi_\phi^2(\tau) + \Psi_r^2(\tau) + \Psi_z^2(\tau) \end{aligned}\right\} \tag{25}$$

Setting equation (23) equal to equation (24) yields

$$\left[a(\tau) + b(\tau)t + c(\tau)t^2\right]^{\frac{1}{2}} = \frac{T/m_0}{1 - \frac{T/m_0}{g_e I_{sp}} t} \tag{26}$$

For specified initial and terminal conditions and thrust level, this equation must be satisfied at any time by a burning time τ. In particular, at $t = 0$, equation (26) becomes

$$\left[a\left(\tau_0\right)\right]^{\frac{1}{2}} = T/m_0$$

When the result is squared, the relation to be solved for the constant-thrust burning time is

$$\alpha_\phi^2\left(\tau_0\right) + \alpha_r^2\left(\tau_0\right) + \alpha_z^2\left(\tau_0\right) = \left(\frac{T}{m_0}\right)^2 \tag{27}$$

Equation (27) is not sufficient to determine the burning time for constant thrust since the thrust level T cannot be arbitrarily specified if six terminal conditions are to be satisfied. If equation (26) is evaluated at the current time t and the burning time τ, two equations result which may be solved simultaneously by iterative means for burning time and the constant thrust level.

For this case, the problem involves 14 unknowns (3 position components, 3 velocity components, 6 acceleration parameters, burning time, and thrust level). These 14 unknowns are completely determined since 6 terminal conditions are specified; 2 constraining relations are introduced to solve for burning time and thrust level; and there are 6 equations of motion with initial conditions.

These values of burning time and thrust are those required to meet the specified end conditions using constant thrust for the approximate problem. As pointed out in a subsequent section, the required values of thrust and burning time will vary slightly during descent or ascent (experience indicates that the thrust varies only a few percent in magnitude). The acceleration parameters, equations (17) to (22), may be calculated by using this value of τ, and then the trajectory may be calculated from equations (10) to (15).

Constant-Pitch-Angle Constraint

The pitch angle θ is found by dividing equation (4) by (5) to obtain

$$\tan\theta = \frac{\alpha_r + \Psi_r t}{\alpha_\phi + \Psi_\phi t} \tag{28}$$

The time derivative of θ is

$$\dot{\theta} = \frac{\alpha_\phi \Psi_r - \alpha_r \Psi_\phi}{\left(\alpha_\phi + \Psi_\phi t\right)^2}\cos^2\theta \tag{29}$$

For θ to be constant, $\dot{\theta} = 0$, so equation (29) yields (in addition to the trivial solutions, $\theta = 90°$ and $270°$),

$$\alpha_\phi \Psi_r - \alpha_r \Psi_\phi = 0 \tag{30}$$

The solution of this equation for burning time will insure a trajectory with a constant pitch angle. The thrust magnitude is, in general, variable under this constraint.

Relaxation of the Range Constraint

For some cases, it may be more useful to allow the range X_τ to be free. For example, if the range is constrained, the required thrust might be greater than the capability of the spacecraft's engines. Relaxing this end condition will permit the thrust to be specified in the constant thrust mode.

In this case there are only seven equations, the six equations of motion and one equation of constraint (equation (26) evaluated at current time t), which insure constant thrust. There are also only five terminal conditions specified, so there may be only 12 unknowns (3 position components, 3 velocity components, 5 acceleration parameters, and burning time). One of the acceleration parameters may be chosen arbitrarily. It is found to be convenient to set Ψ_ϕ equal to 0. Equation (18) may then be solved for X_τ.

$$\Psi_\phi = 0 = \frac{6}{\tau^2}\frac{r_0}{R}\left[\frac{2}{\tau}\left(X_0 - X_\tau\right) + \dot{X}_0 + \dot{X}_\tau\right]$$

$$X_\tau = X_0 + \frac{\tau}{2}\left(\dot{X}_0 + \dot{X}_\tau\right) \tag{31}$$

Equation (28) for the tangent of the pitch angle becomes linear in t

$$\tan\theta = \frac{\alpha_r}{\alpha_\phi} + \frac{\Psi_r}{\alpha_\phi}\,t$$

In reference 1, it is shown that this is the form that the bilinear tangent law takes when the final range is allowed to be free.

Substitution of equation (31) into equation (17) yields

$$\alpha_\phi = \frac{r_0}{R\tau}\left(\dot{X}_\tau - \dot{X}_0\right) \tag{32}$$

The equations for the acceleration parameters α_r and Ψ_r become

$$\alpha_r = \frac{\mu}{r_0^{\,2}} - \frac{r_0}{R^2}\dot{X}_0^{\,2} + \frac{1}{\tau}\left(\dot{h}_\tau - \dot{h}_0\right) - \frac{C_2}{2}\tau - \frac{C_3}{3}\tau^2 \tag{33}$$

$$\Psi_r = -2\alpha_\phi \frac{\dot{X}_0}{R} + \frac{12}{\tau^3}\left(h_0 - h_\tau\right) + \frac{6}{\tau^2}\left(\dot{h}_0 + \dot{h}_\tau\right) - C_3\tau \qquad (34)$$

The acceleration parameters α_z and Ψ_z are unchanged.

USE OF THE SIMPLIFIED TWO-POINT BOUNDARY-VALUE SOLUTION IN GUIDING A SPACECRAFT IN A GRAVITATIONAL FIELD

The solution that has been presented is valid only within the approximations made; that is, for small changes in altitude and in the z direction. Therefore, there would be inherent errors if the equations were used to predict the position and velocity of a spacecraft that is thrusting for long distances over a spherical attracting body. These errors are minimized, however, if the equations are solved again after a small interval of flight time. The initial conditions and the burning time are changed after each such interval. The new initial conditions are obtained from the numerical integration of the exact equations (1), (2), and (3) with the use of the control variables θ, β, and A. These control variables are generated from the same set of acceleration parameters through equations (4), (5), and (6) that were used in the simplified solution given in equations (10) to (15). The smaller the time interval between the updating of the initial conditions the more closely the trajectory will meet the prescribed end conditions.

The performance of the acceleration-parameter relations as guidance equations has been evaluated by a simulation programed on an IBM 7094 digital computer. A trajectory program which numerically solves equations (1), (2), and (3) to predict the position and velocity of the spacecraft at any time is a basic component of the simulation. A guidance program which continually computes the acceleration and its direction by solving equations (17) to (22) is the other basic component of the simulation. The computation procedure is depicted graphically in figure 2.

The problem of combining the two programs for variable thrust is straightforward. Both sets of equations are given the same initial conditions. The final conditions are also substituted into the acceleration parameter relations given in equations (17) to (22). The acceleration parameters α_ϕ, Ψ_ϕ, α_r, Ψ_r, α_z, and Ψ_z are computed with these conditions and are used to calculate θ, β, and A from equations (4), (5), (6), and (24) for the first step. The output, X, $\dot{X}$, h, $\dot{h}$, z, $\dot{z}$, of the first integration step is then used as the initial conditions in equations (17) to (22) to compute θ, β, and A for the next integration step. The calculation thus proceeds to the final conditions of the problem.

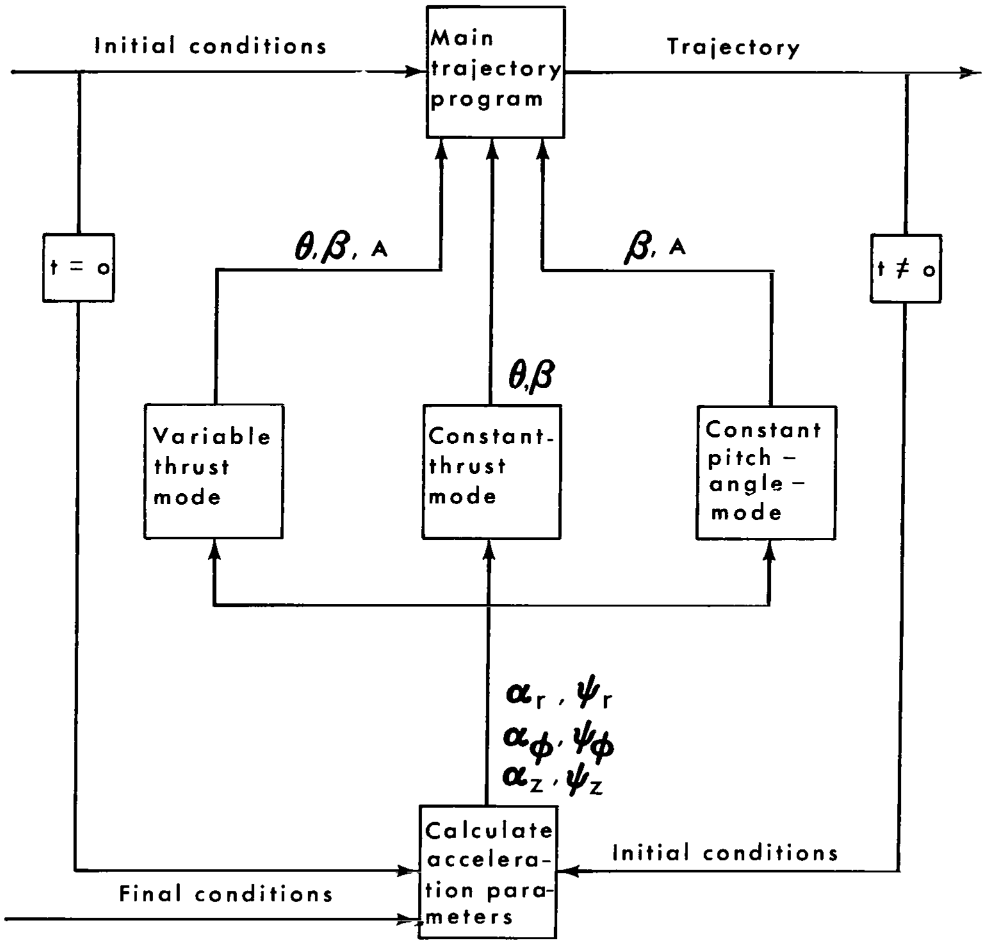


Figure 2.- Block diagram illustrating guidance simulation

If the terminal range is specified, a nearly constant-thrust trajectory may be generated by solving equation (26), evaluated at $t = 0$ (eq. (27)) and burning time τ, simultaneously by using updated initial conditions for thrust and burning time after each integration step. The updated value of burning time is used in equations (17) to (22), along with the updated initial conditions, to calculate θ and β as in the variable thrust case. In test cases that were run, the thrust magnitude varied only about 3 percent.

If the range is allowed to be free, the thrust level is specified at some constant value, and equation (27) is solved for burning time using the updated initial conditions after each integration step.

For constant-pitch-angle trajectories the burning time is updated after each time step by solving equation (30) and then using the updated burning time and initial conditions to update A and β.

As the terminal conditions are approached, the burning time, which is essentially the time to go before cut-off, decreases and reduces to zero at cut-off. The acceleration parameters given by equations (17) to (22) approach infinity as burning approaches zero. This difficulty was overcome in the examples presented by holding the values of the acceleration parameters constant for the last few integration steps.

The two examples presented in figure 3 and table I illustrate some of the results of the simulation. The first example (fig. 3) is a portion of a constant thrust, range-free, in-plane, lunar-landing maneuver from an altitude of 50 000 feet to 10 000 feet with a thrust-to-initial-weight ratio of 0.4. The linear-acceleration guidance solution is compared with a calculus-of-variations minimum-fuel solution. Even though the trajectories are not the same, being quite different in altitude and vertical velocity, the desired final conditions are the same, and the burning times are very nearly equal. In both cases, the amount of fuel burned was for all practical purposes the same. The total burning time was 300.9 seconds. The acceleration parameters were held constant during the last 14 seconds.

The second example (table I) is a portion of a lunar landing from an altitude of 10 000 feet to 1000 feet in which a constant pitch angle of 139.69° is used. In this example, it was specified that the final position must be 10 000 feet away from the initial plane of the motion. All of the six end conditions were within reasonable tolerances of those specified. The final altitude was only 3 feet higher than that specified, and the horizontal and vertical velocities were each approximately 2 ft/sec greater than that specified. The fuel burned was 0.1028 of the initial weight. The total burning time was 94.20 seconds. The acceleration parameters were held constant during the last 0.2 second.

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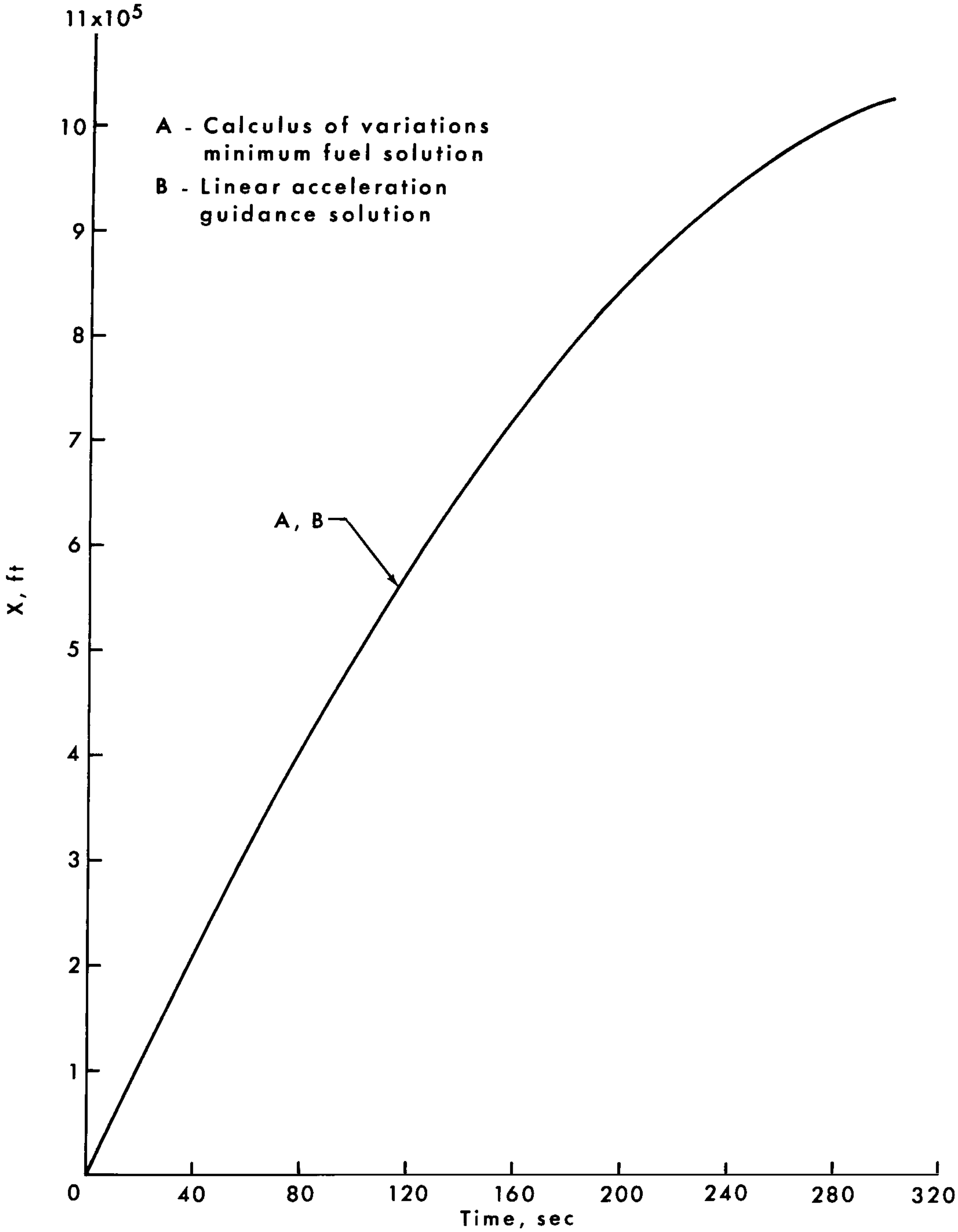
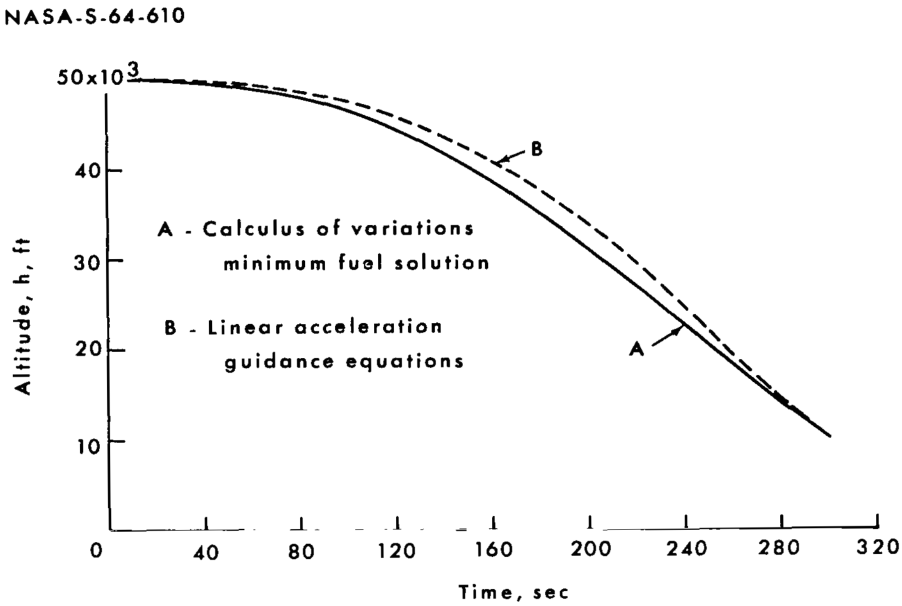
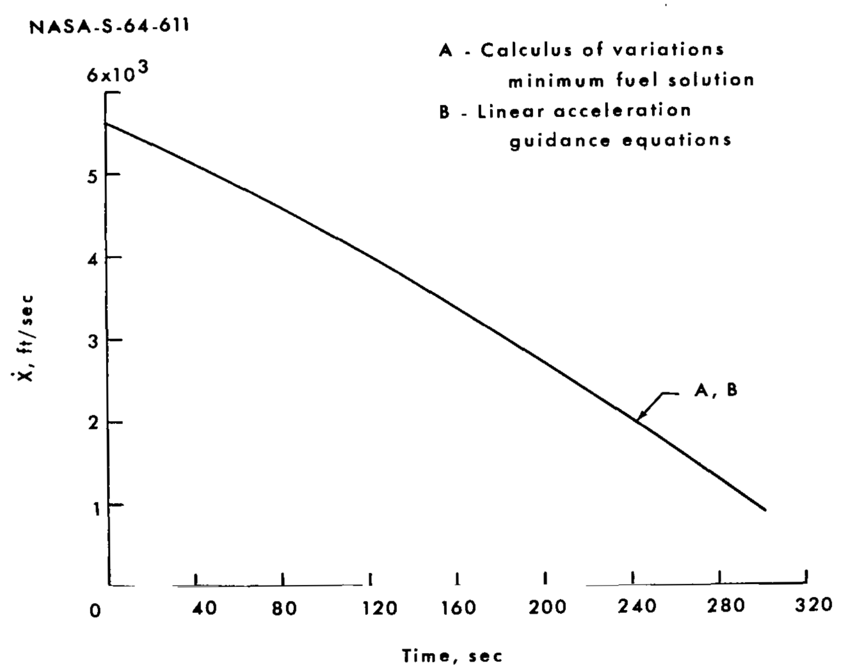


(a) Range as a function of time

Figure 3.- Comparison of minimum fuel and linear acceleration guidance solutions for a lunar descent problem $\left(T/W_0 = 0.4;\ I_{sp} = 315\ \text{sec}\right)$.



(b) Altitude as a function of time



(c) Horizontal velocity as a function of time

Figure 3.- Continued

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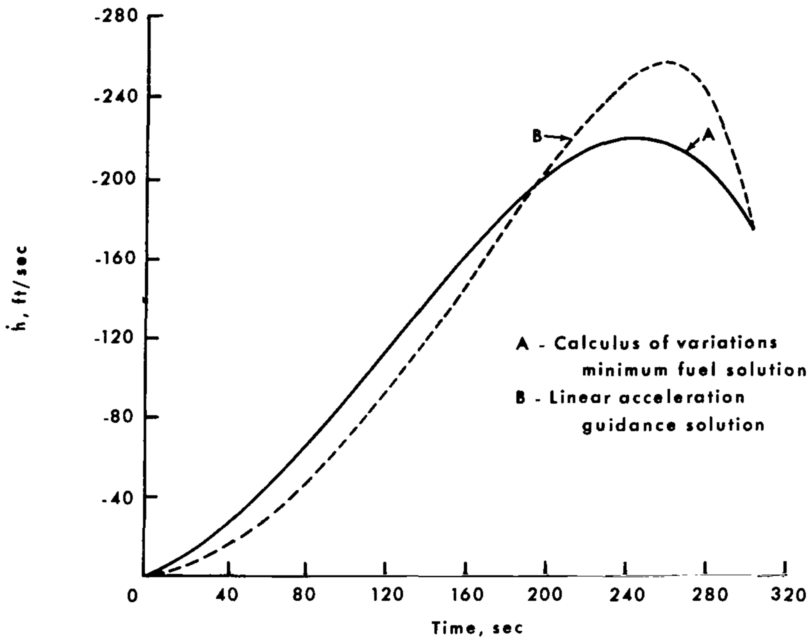

(d) Vertical velocity as a function of time

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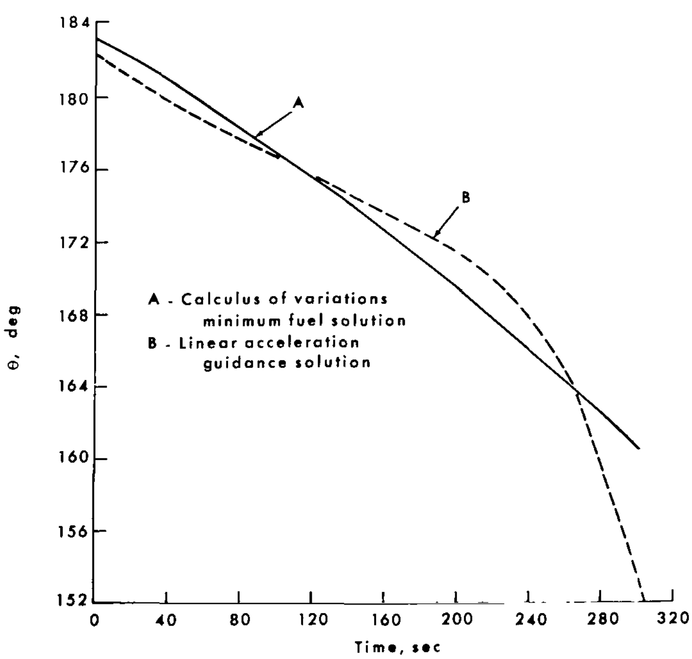

(e) Pitch angle as a function of time

Figure 3.- Concluded.

TABLE I.- OUT-OF-PLANE CONSTANT-PITCH-ANGLE DESCENT FROM ALTITUDE OF 10 000 FEET TO 1000 FEET ABOVE LUNAR SURFACE

$$\left[\theta = 139.69^\circ;\ \tau = 94.20 \text{ sec};\ I_{sp} = 315 \text{ sec}\right]$$

	Initial conditions	Final conditions	
	Specified values	Integrated values	Specified values
X, ft	0	48 000.8	48 000.0
h, ft	10 000	1003.0	1000.0
z, ft	0	10 000.9	10 000.0
$\dot{X}$, ft/sec	891.47	101.93	100.0
$\dot{h}$, ft/sec	174.72	2.02	0
$\dot{z}$, ft/sec	0	0.78	0

CONCLUDING REMARKS

A guidance scheme for landing and launch maneuvers in a central force field has been investigated and simulated on a digital computer. The flexibility of the guidance scheme has been demonstrated by two examples. The first example is a lunar-landing maneuver from an altitude of 50 000 to 10 000 feet with a constant thrust-to-initial-weight ratio of 0.4. This case was compared with a calculus-of-variations fuel-optimum trajectory, between the same end points, and the fuel burned was the same for both methods. The second is a lunar-landing maneuver from an altitude of 10 000 to 1000 feet with variable thrust and a constant pitch angle of 139.69°. The specified end conditions were attained in both examples.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, December 4, 1964

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